Knowsl@y Council

Knowsley LA Policy

On Development of Calculation in Key Stages 1 and 2

Appendix 1: **Criteria for readiness for formal written methods of addition and subtraction.**

Appendix 2: **NNS Framework references for pencil and paper procedures (+ and -) (x and ÷).**

Appendix 3: **Approaches to calculation: questions answered.**

Education & Lifelong Learning

Huyton with Roby Mathematics policy on the development of calculation

- Each teacher will develop the mental strategies for their children appropriate to the year group with modifications for the more able and the less able. (see NS Primary Framework)

- Within the school there will be a development of recording mental strategies, which allows the children to make their thinking visible.

- Initially children will be given opportunities to record just using the numerals or pictorial representations without the use of signs for operations or the equals sign.

- Children will then develop understanding of plus and minus and equals signs by using appropriate contexts.

- Children may record on plain paper in the foundation stage, Year 1 and Year 2, although, if appropriate, large, squared paper ($2cm^2$) may be introduced.

- Children may record on squared paper in Year 2 Year 3 Year 4 Year 5 and Year 6. Although continuing to record on plain paper allows children to develop their setting out of calculation methods. They do not have squared paper in the QCA tests.

- Throughout the equals sign will be used accurately, e.g. 3+4=7+8=15 is not appropriate, but 3 + 4 = 7
  7 + 8 = 15 is appropriate.

- Children will have experience of a range of word sentences in a diversity of forms.
  e.g. 3 + 4 + 5 = 12 7 = 4 + 3 3 + 4 = 5 + 2

- Children will have the opportunity to experience missing box problems with the box being in any of the three positions.

  e.g. $7 + \square = 12$ $\square = 7 + 5$ $12 = 7 + \square$

- Children will use different methods of recording mental strategies which include number sentences, the empty number line and arrow diagrams.

  (Examples of empty number lines are shown overleaf).

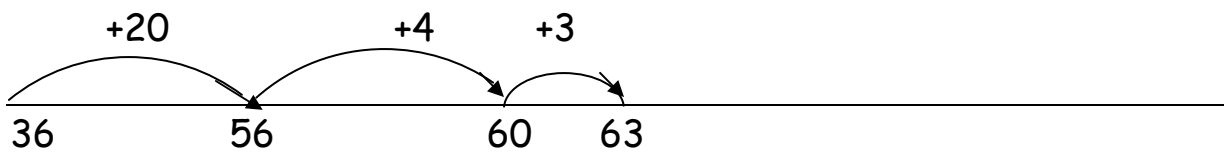
Empty number lines

……

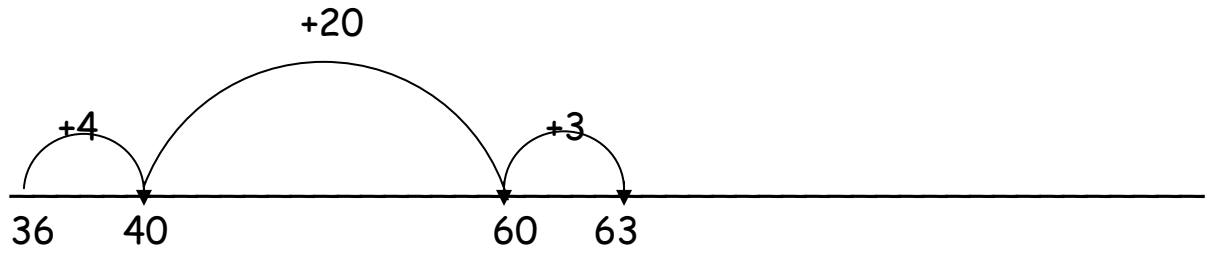
Empty number lines like these are sketched by children themselves. They are used to support mental calculations and do not need to be drawn to scale.

Addition 36+27

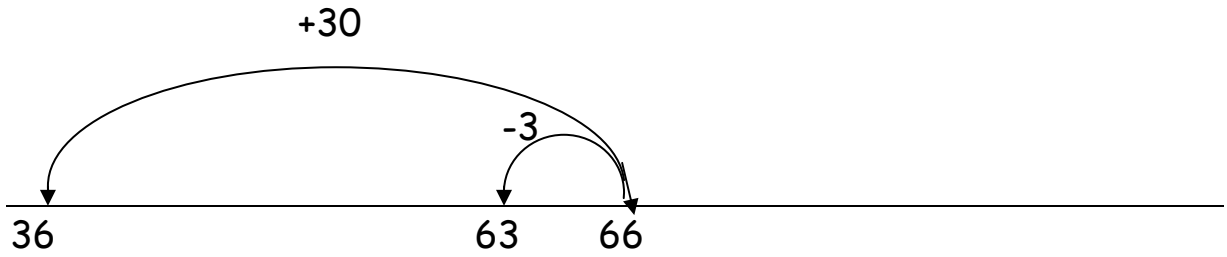
- Jumping in tens and ones



- Jumping to the next multiple of ten

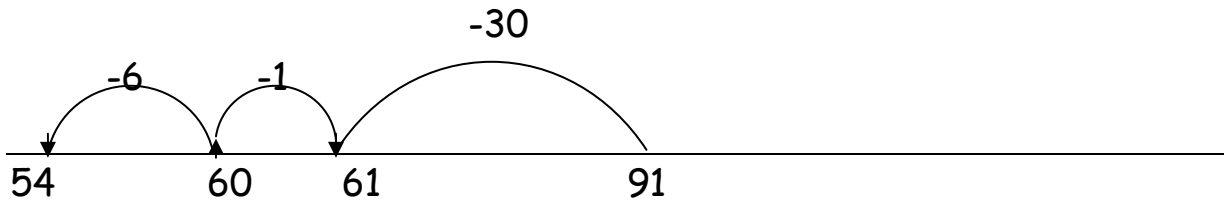


- Overjumping

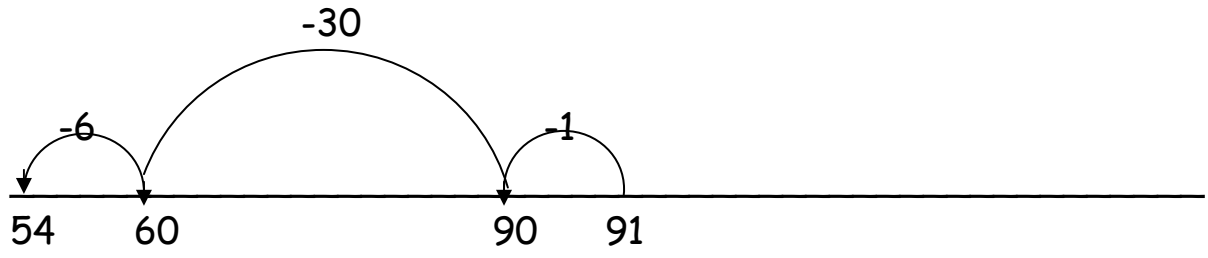


Subtraction 91 -37

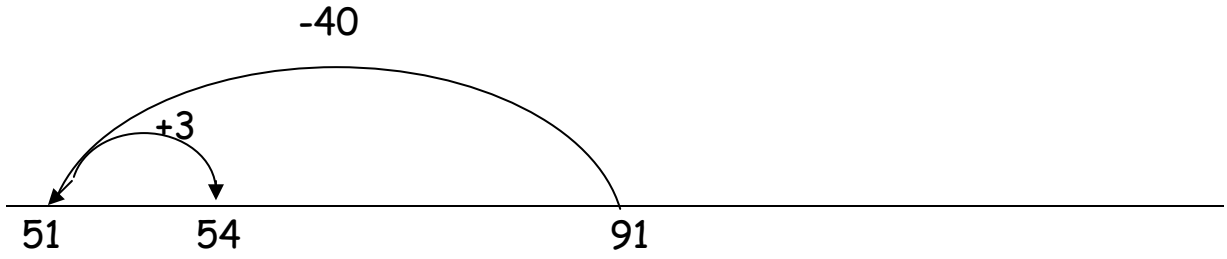
- Jumping in tens and ones



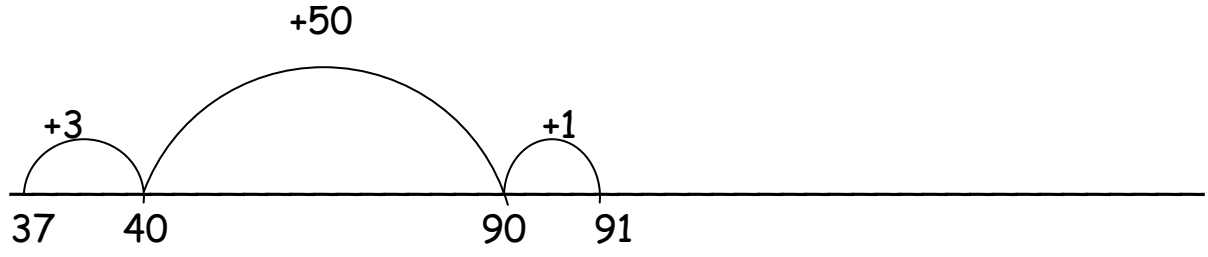
- Jumping to the next multiple of ten



- Overjumping

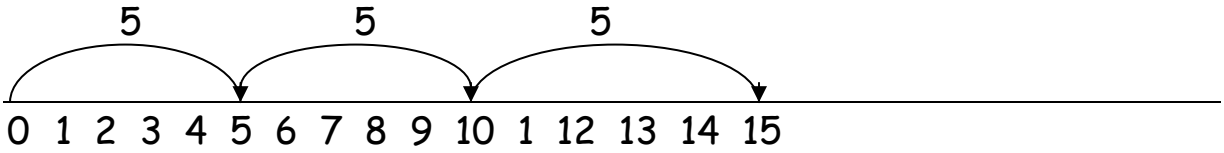


Finding a small difference or counting on



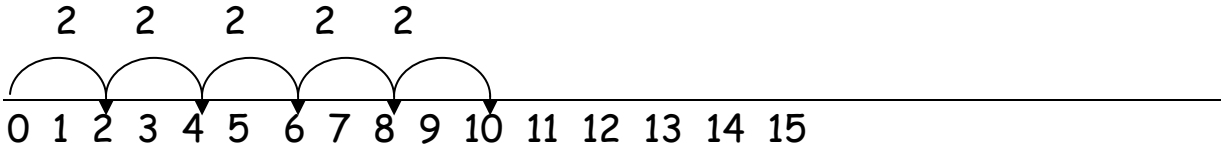
Multiplication

- 5 X 3



Division

- How many 2s are there in 10?



The aim for mental calculation is that by the end of Key Stage 2, pupils have a repertoire of methods for each of the operations including partitioning and recombining.

Expanded written calculation for addition and subtraction will be developed when the child can:

- Add and subtract two single digit numbers quickly
- Add and subtract multiples of ten quickly
- Partition numbers into tens and units
- Add and subtract two digit numbers mentally and record the method informally.

The aim for written calculation is that by the end of Key Stage 2, pupils can use a compact written method for each of the four operations. However, some children may still be using an expanded format in one or more of the operations.

Stages in teaching addition

STAGE ONE:
Mental method, using partitioning.

47 + 76 = (40 + 70) + (7 + 6)
or
47 + 76 = (47 + 70) + 6

STAGE TWO:
Demonstrate the expanded vertical layout of a calculation, showing the addition of the tens and the addition of the ones separately. Use an example which children can already do mentally, so they can see how it works. Discuss how adding the least significant digit first still gives the same answer.

47	47
+ 76	+76
110	13
13	110
123	123

STAGE THREE:
Extend to bigger numbers. Ask children to estimate first.

(Example: 368 + 493. Estimate: 370 + 500 = 870)

Emphasise to the children that this vertical layout is simply a more organised way of writing their mental calculations. Ask them to check if the answer is sensible. You may start with either the most or least significant digit first.

368	368
+493	+493
700	11
150	150
11	700
861	861

STAGE FOUR:

Show children the compact layout alongside the expanded format. Say that, when they are using this quick way of writing a calculation, they should get in the habit of adding the least significant digit first at Stage Four. Again, start with a calculation that they can do mentally so that they can easily follow the procedure and appreciate how it works. Introduce calculations which involve one exchange and then two or more. Still encourage children to estimate first.

One exchange: (underneath the line)

47	47	368	368	368	368
+26	+26	+423	+423	+491	+491
13	73	700	791	9	859
60	1	80	1	150	1
73		11		700	
		791		859	

Two exchanges:

47	47	368	368
+76	+76	+493	+493
13	123	700	861
110	1 1	150	1 1
123		11	
		861	

STAGE FIVE: -

Use these layouts with bigger numbers and decimals. Children may need to revert to more expanded layouts, initially.

<u>The following language will be used for addition</u>:

Exchange will be used for the movement from column to column.
Numbers will always referred to by their value and not as digits.

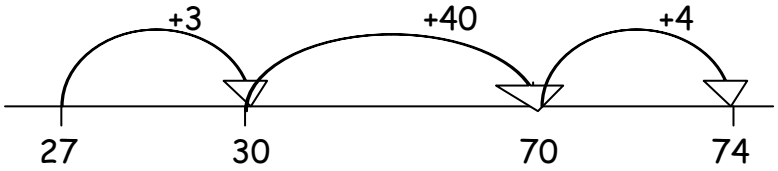
Stages in Teaching Subtraction

The following expanded form will be used for subtraction:

STAGE ONE:

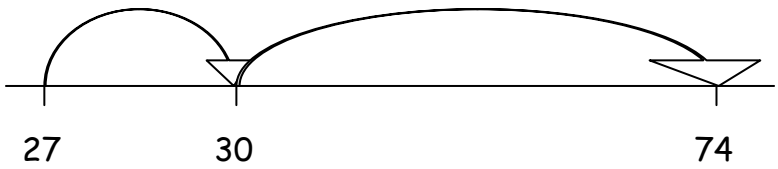
Show children the vertical layout for a calculation they can do mentally. Link the steps to those on an empty number line using post-it notes to support.

E.g. 74 – 27



74
-27
+3 (⇒ 30)
+40 (⇒ 70)
+ 4 (⇒74)
+47

or



74
- 27
+3 (⇒ 30)
+44 (⇒ 74)
+47

Although complementary addition can continue with three or even four digit numbers we think it is more helpful to introduce the expanded format for decomposition at the next stage.

Example of expanded subtraction calculations

STAGE TWO: 81-57

81	80 + 1		70 + 11	
- 57	= 50 + 7	=	50 + 7	
			20 + 4	= 24

Three digit numbers

563 – 241

500 + 60 + 3	Leading to:	563
- 200 + 40 + 1		- 241
300 + 20 + 2		322

STAGE THREE : 563 – 248

500 + 60 + 3
-200 +40 + 8

		50 13		5 1
500 + 50 + 13	or	500 + 60 + 3	Leading to:	563
-200 + 40 + 8		-200 + 40 + 8		- 248
300 + 10 + 5		300 + 10 + 5		315

STAGE FOUR : 563 – 278

500 + 60 + 3
- 200 + 70 + 8

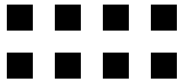
		150		
		400 50 13		4 15 1
400 + 150 + 13	or	500 + 60 + 3	Leading to:	563
- 200 + 70 + 8		- 200 + 70 + 8		- 278
200 + 80 + 5		200 + 80 + 5		285

Language used for subtraction:

Exchange will be used to explain the decomposition of the number. Emphasis will be placed on the number being the same but partitioned differently.
Again numbers will always be referred to by their value and not as digits.

Stages in Teaching Multiplication

In multiplication children will develop an understanding of the different aspects of multiplication.
Repeated addition: for example, 5 added together 3 times is 5+5+5, or 3 lots of 5, or 5 x 3 (five multiplied by three).
Describing an array: for example,



4 x 2 =8
2 x 4 = 8

Children will begin to recognise from arranging arrays that multiplication can be done in any order, e.g. 4 x 2 and 2 x 4 give the same product.
The representation of multiplication as an array will be used to link the mental strategies children are using to the written calculation.
The array will be used to demonstrate the distributive law.
The array will then be replaced by a grid and thus lead to the grid method as described in the framework.

STAGE ONE:
Recording mental multiplications in a number sentence using the x and = signs. Recall of multiplication table facts and understanding multiplication is the inverse of division.

Mental method, using partitioning for TU by U.

38 x 7 = (30 x 7) + (8 x 7)

STAGE TWO:
Grid layout, expanded working.

X	30	8	
7	210	56	266

STAGE THREE:
Extended to bigger numbers.

Example: 56 x 27.
Estimate: 1800 because 60 x 30 = 1800
56 x 27 = (50 + 6) x (20+ 7)

x	50	6	
20	1000	120	1120
7	350	42	392
			1512

STAGE FOUR:
Extend to bigger numbers and decimals.

Example: 23.5 x 12.
Estimate: 25 x 10 = 250
23.5 x 12 = (20 + 3 + 0.5) x (10 = 2)

x	20	3	0.5	
10	200	30	5	235
2	40	6	1	47
				282

STAGE FIVE:
Vertical format, expanded working.
Again, start with a calculation the children can do mentally. Show links with the grid method.

```
  38
 x 7
 210   (30 x 7 = 210)
  56   (8 x 7 =  56)
 266
```

Link to grid method:

x	30	8	
7	210	56	266

```
  56
 x 27
1000   (50 x 20 = 1000)
 120    (6 x 20 =  120)
 350   (50 x  7  =  350)
  42    (6 x  7  =   42)
1512
```

Link to grid method:

x	50	6	
20	1000	120	1120
7	350	42	392
			1512

STAGE SIX:

Vertical format, compact working.

```
    38
x    7
   266
    5
```

Link to grid method:
(Children may need to use jottings to support the individual multiplications in this method.)

```
    56
x   27
  1120   (56 x 20)
   392   (56 x  7)
  1512
  1
```

x	50	6	
20	1000	120	1120
7	350	42	392
			1512

Stages in Teaching Division

In division children will be introduced to both sharing and grouping, for example 35 ÷ 5 as "how many 5s make 35 ?".
Grouping will then be used to lead children on to repeated subtraction.
This will be developed through the use of appropriate contexts to encourage children to use a repeated subtraction method which over an extended period of time will be refined into the most efficient way of using repeated subtraction.
Recall of division facts should be learnt alongside multiplication facts.
Children should understand division as being the inverse of multiplication.

Recording simple mental divisions in a number sentence using ÷ and = signs. Give a whole number remainder when one number is divided by another, for example work out that 16 ÷ 3 = 5 remainder 1.
Make sensible decisions about rounding up or down after division, for example 46 ÷ 5 is 9 remainder 1, but whether the answer should be rounded down to 9 or up to 10 depends on the context:
I have £46. Tickets cost £5 each. I can only buy 9 tickets.
There are 46 children. A table seats 5 children. 10 tables are needed to seat all the children.
Partitioning and **factorising** are mental division strategies to be developed.

Partitioning:

Example: 78 ÷ 6 =
(60 + 18) ÷ 6 = 10 + 3
= 13

Example: 75 ÷ 5 =
60 + 15 ÷ 5 = (60 ÷ 5) + (15 ÷ 5)
12 + 3 = 15

Factorising

40 x 15 =		600 ÷ 15 =
40 x 5 x 3 =		(600 ÷ 3) ÷ 5 =
200 x 3 = 600	leads to	200 ÷ 5 = 40

Example: 280 ÷ 20=
280 ÷ (10 x2) =
(280 ÷ 10) ÷ 2 =
28 ÷ 2 = 14

STAGE ONE:

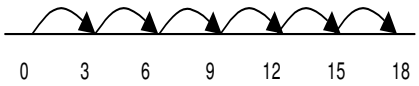
Show an example which can be done mentally, model division by repeated addition and repeated subtraction for all stages – children often find repeated addition easier to understand. Questions requiring grouping and sharing.

Model on a numberline first with individual jumps of the divisor, and then chunking.

Progress to modelling on a vertical numberline, individual jumps of the divisor and then chunking.

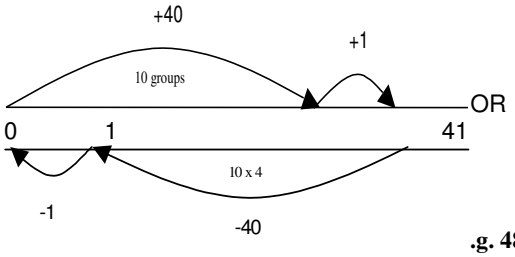
Model chunking with children listing partial tables at the side of significant numbers. See below example.

Grouping - How many 3's make 18?

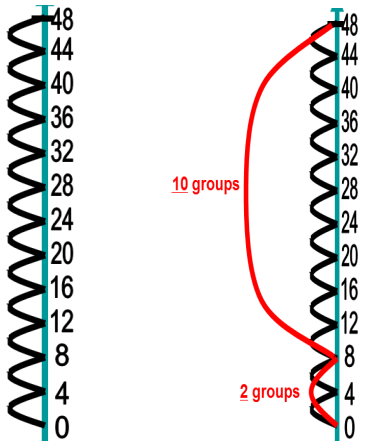


Remainders

41 ÷ 4 = 10 r1



.g. 48 ÷ 4



Example: 97 ÷ 9

9) 97

- 90 (9 x 10)
7

Answer: 10 R7

STAGE TWO – record vertically after modelling on horizontal and vertical numberline, chunking and partial tables recorded:

72 ÷ 5 lies between 50 ÷ 5 = 10 and 100 ÷ 5 = 20

72
- 50 (10 groups) or (10 x 5)
22
- 20 (4 groups) or (4 x 5)
2

1x	5
2x	10
5x	25
10x	50

Answer : 14 remainder 2

Children should write partial tables in a menu box. This will help them in identifying the largest group they can subtract in one chunk.

Now apply to larger numbers.
Ask the children to estimate the answer first.
Example: 196 ÷ 6
Estimate: 200 ÷ 5 = 40

6)196
- 60 6 x 10
136
- 60 6 x 10
76
- 60 6 x 10
16
- 12 6 x 2
4
Answer: 32 R4

Contract as children's Mental strategies improve.

6) 196
- 120 6 x 20
176
- 60 6 x 10
16
- 12 6 x 2
4
Answer: 32 R4

Need to have an understanding of estimations.

STAGE THREE:
Estimating can have two purposes when doing a division calculation. The more familiar is to check the answer after the calculation. The other purpose is to help chose a starting point to divide by. (Estimating multiples of the divisor.)

For 196 ÷6, the first estimate was 40. But 40 isn't a good first divisor, as 6 x 40 = 240, which is more than 196.
So children should find a starting point which is less than 40, such as 30.

As children's estimates improve, the number of steps will decrease.

Estimate to find a good divisor: use 30 because 6 x 30 = 180

```
6 )196
   - 180   6 x 30
     16
   -  12   6 x 2
     4
```

Answer: 32 R4

Note that this most compact layout is similar to that of conventional long division with zero as a placeholder.

STAGE FOUR:
Extend to bigger numbers and decimals.

Appendix 1

Criteria for readiness for formal written methods of addition and subtraction

- Do the children know addition and subtraction facts to 20?
- Do they understand place value and can they partition numbers into hundreds, tens and ones.
- Can they add at least three single-digit numbers mentally?
- Can they add and subtract any pair of two-digit numbers mentally?
- Can they explain their mental strategies orally and record them using informal jottings?
- Do they use and apply the commutative and associative laws of addition?

Criteria for readiness for formal written methods of multiplication and division

- Do the children know the 2, 3, 4, 5 and 10 times-tables and the corresponding division facts?
- Do they know the result of multiplying by 0 and 1?
- Do they understand place value?
- Do they understand 0 as a place holder?
- Can they multiply two-and three-digit numbers mentally by 10 and 100?
- Can they use their knowledge of all the multiplication tables to approximate products and quotients using powers of 10?
- Can they double and halve two-digit numbers mentally?
- Can they use multiplication facts that they do not know?
- Can they explain their mental strategies orally and record them using informal jottings?
- Do they use commutative law, the associative law and the distributive law for multiplication?

Appendix 2

Calculations, pencil and paper procedures (+ and -) – Reception to Y6

Reception	
Year 1	**Year 2**
Year 3	**Year 4**
42-45 Pencil and paper procedures (+ and -) 43, 45 Use informal pencil and paper methods to support, record or explain HTU ± HTU. Begin to use column addition and subtraction for HTU ± TU where the calculation cannot easily be done mentally.	48-51 Pencil and paper procedures (+ and -) 48, 50 Use informal pen and paper methods to support, record or explain additions / subtractions. **Develop and refine written methods for: column addition and subtraction of two whole numbers less than 1000 and addition of more than two such numbers;** Money calculations (for example, £7.85± £3.49)
Year 5	**Year 6**
48-51 Pencil and paper procedures (+ and -) 49, 51 Use informal pencil and paper methods to additions / subtractions. **Extend written methods to:** **column addition/subtraction of two integers less than 10000;** addition of more than two integers less than 10000; addition or subtraction of a pair of decimal fractions, both with one or both with two decimal places (eg £29.78 + £53.34).	48-51 Pencil and paper procedures (+ and -) 49, 51 Use informal pencil and paper methods to support, record or explain additions/subtractions. **Extend written methods to column addition and subtraction of numbers involving decimals.**

Calculations, pencil and paper procedures (x and ÷) – Reception to Y6

Reception	
Year 1	**Year 2**
Year 3	**Year 4**
	66-69 Pencil and paper procedures (x and ÷) 66, 68 Approximate first. Use informal pencil and paper methods to support, record or explain multiplications and divisions. Develop and refine written methods for TU x U, TU ÷ U.
Year 5	**Year 6**
66-69 Pencil and paper procedures (x and ÷) 67, 69 Approximate first. Use informal pencil and paper methods to support, record or explain multiplications and divisions. **Extend written methods to:** **Short multiplication of HTU or U.t by U;** **Long multiplication of TU by TU;** **Short division of HTU by U (with integer remainder).**	66-69 Pencil and paper procedures (x and ÷) 67, 69 Approximate first. Use informal pencil and paper methods to support, record or explain multiplications and divisions. **Extend written methods to:** Multiplication of ThHTU x U (short multiplication); **Short multiplication of numbers involving decimals;** **Long multiplication of a three-digit by a two-digit integer;** Short division of TU or HTU by U (mixed-number answer); Division of HTU by TU (long division, whole-number answer); **Short division of numbers involving decimals.**

Appendix 3

Approaches to calculation: questions answered

Below are questions teachers most frequently ask about teaching calculations and some suggested answers.

- **Do all mental methods have to be taught to all pupils?**

It is useful to know 'special case' mental methods: (e.g. to add 39 by adding 40 and taking away 1, or to multiply by 25 by multiplying by 100, then dividing by 4). Some children work out these methods for themselves but others need to be taught. Teachers need to find out which strategies children are using by asking them to explain their methods. They should then help children to refine their methods. To make them more efficient.

- **How should we progress from mental to written methods?**

Standard written methods are based on steps which are done mentally. It is important that the mental strategies required to tackle written calculations are taught and mastered first. Informal written methods can provide an effective transition to standard written methods. This transition should not be hurried and not all children will be ready for standard methods at the same time.

- **How should we relate mental methods to standard written methods?**

Standard written methods are based on steps which are done mentally. It is important that the mental strategies required to tackle written calculations are taught and mastered first. Informal written methods can provide an effective transition to standard written methods. This transitions should not be hurried and not all children will be ready for standard methods at the same time.

- **How should we relate mental methods to standard written methods?**

Mental methods are often particular to the numbers in a calculation.
Working out 29 x 7 mentally can involve a different strategy from that used for 228 x 8. We do not apply the same mental method all the time. The goal for written methods is to give children a general or standard method that can be used for all cases when a mental method is inappropriate. Children will use their mental recall skills, and will draw on their understanding of place value and the laws of arithmetic, but there may be no direct link with the mental strategies that they have been using.

For example, many people will add numbers mentally by adding the most significant digits first, yet column addition is conventionally carried out by adding the least significant digits first. But children who have a good grounding in the partitioning of numbers, and who can recall addition facts quickly and confidently, have little difficulty in transferring from the former to the latter when they cannot do the calculations mentally.

- **What should we do with older pupils who have never met informal written methods?**

If the children can do calculations accurately and consistently, they should continue to use formal methods. Able children might be stimulated by the opportunity to explore other methods, such as Chinese or Egyptian multiplication (described in some teachers' reference books.) Children who make errors with formal methods should be introduced to informal methods, with the explanation that it may help them to understand what they are doing and become more successful. With these children, return to formal methods only when they can use an informal method accurately and explain what they are doing.

- **Do calculators have place below Year 5?**

Other ways of calculating should come first. However, calculators can be a useful teaching aid, for example, to develop understanding of the number system or estimation skills. Look at page 8 in the introductory section of the Framework. This discusses the role of calculators.